Technical Companion to:

Sharing Aggregate Inventory Information with Customers: Strategic Cross-selling and Shortage Reduction

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This technical companion serves as a supplementary material. We relax the assumptions of the model in the paper and extend out main results to the following settings: (1) the total inventory level is more than the total demand in the market, (2) the search cost is heterogenous across heterogeneous customers, and (3) the allocation rule is efficient or inefficient. In addition, we provide the comparative statics analysis of how the aggregate information disclosure decision is impacted by search cost when customers have heterogeneous tastes.

1 Limited Capacity

In the paper, we assume the total demand in the market exceeds the total inventory carried by the seller, i.e., $N > M$. We next relax this assumption and show that the key finding in the paper remains when there is more inventory than total demand. We next show that when the inventory is somewhat limited, i.e., $(1-q)M < N < M$, the main result remains: aggregate inventory information disclosure sustains as an equilibrium. When the seller carries a lot of redundant inventory, i.e., the amount of desired product exceeds the total demand, information sharing strategy fails to alter any outcome because all customers will visit the store no matter what information sellers choose to disclose to customers.

The first condition that ensures the unfavorable scenario to choose the aggregate disclosure strategy is $N_u^{Full} < N$. As the inventory increases, the out-of-stock risk declines, and it is possible that all customers visiting the store. Avoiding this situation for the unfavorable scenario under full information disclosure provides the seller the incentive to attract more demand. Otherwise, if all the customers in the market visit the store, all demand can be satisfied, and thus the seller has no incentive to pool information. The second condition for the favorable scenario is $N^{Agg} = N$, which requires all the potential customers in the market to visit the store under aggregate information. As aggregate information disclosure hides the good news from customers in the favorable scenario, the number of incoming customers in the favorable scenario will decrease or stay the same. If the number decreases, since all incoming demand can be satisfied (because there is more inventory than demand), this means the seller in the favorable scenario incurs a higher shortage penalty cost from information aggregation and will prefer full information disclosure. Therefore, the condition for the favorable scenario to not lose profit is $N^{Agg} = N$.

To summarize, the intuitions behind the two conditions are the same as the case discussed in Lemma 1 in the paper. The first condition requires the seller in the unfavorable scenario to not meet all demand under full information and thus she is motivated to use information pooling as a lever to increase sales. The second condition requires the seller in the favorable scenario to meet all demand in the market under aggregate information so as not to jeopardize her profit. We formalize the above discussions in the following proposition.

Proposition 1 *When the shortage penalty cost is zero, and the seller carries more inventory than the total demand in the market, $M > N$, the two conditions for aggregate inventory information disclosure become:*

- *The seller in the unfavorable scenario fails to satisfy all demand under full information, i.e., $N_u^{Full} < N$.*
- *The seller in the favorable scenario can meet all demand under aggregate information, i.e., $N^{Agg} = N$.*

When $qM < N < M$, the region in which the aggregate disclosure is the optimal policy is

$$\theta_H + \frac{(1-q)M(\theta_L - p_L) - Nc}{N - (1-q)M} < p_H < \theta_H + \frac{(q(1-\beta) + (1-q)\beta)\, M(\theta_L - p_L) - Nc}{N - M(q(1-\beta) + (1-q)\beta)}.$$

When $(1-q)M < N < qM$, the region in which the aggregate disclosure is the optimal policy is

$$\theta_H + \frac{(1-q)M(\theta_L - p_L) - Nc}{N - (1-q)M} < p_H < \theta_H + \frac{(N + \beta(1-q)M - \beta N)(\theta_L - p_L) - Nc}{Mq\beta}.$$

When $N < (1-q)M$, all customers will visit the store regardless of the type of information shared by the seller. The seller is indifferent in the types of inventory information disclosure.

2 Heterogeneous Search Cost

Recall for heterogeneous customers, different people have different valuations regarding the products and the valuation v is uniformly distributed over $[0, 1]$. We assume in the paper that the search cost is constant across customers. Recent research in the literature has shown evidence of heterogeneous search cost among customers. For example, Chen et al. (2008) provide some anecdotal evidence of heterogeneous search cost among consumers from Hewlett-Packard marketing research. Moon et al. (2015) also finds empirical evidence of heterogeneity in the consumers' monitoring cost.

In this model, we relax this assumption by considering the search cost is related to the valuation. To be specific, the customers who receives high value from the product might value the time spent on purchasing more. We assume the search cost for type v customer is $v\hat{c}$, where $\hat{c}$ is the unit search cost. We next consider the case with two inventory scenarios that have the same total inventory M. We follow the simplified assumption in Section 5.2 of the paper that all inventory scenarios occur with an equal probability, i.e., $\beta = 0.5$, and the shortage penalty cost is positive but negligible compared with the profit margin for any product.

Let x denote the lowest type of the incoming customer under full information in the unfavorable scenario and y denote the lowest type of incoming customer under aggregate information. Analogous to the marginal equation (1) and (2) in the paper, given the customers of type x and y are indifferent between visiting and not visiting the store, it follows that $P_u^{Full}(\theta_L x - p_L)^+ + \bar{P}_u^{Full}(\theta_H x - p_H)^+ = x\hat{c}$ and $P_u^{Agg}(\theta_L y - p_L)^+ + \bar{P}_u^{Agg}(\theta_H y - p_H)^+ + P_f^{Agg}(\theta_L y - p_L)^+ + \bar{P}_f^{Agg}(\theta_H y - p_H)^+ = 2y\hat{c}$.

For heterogeneous customers with heterogeneous search cost, aggregate information disclosure requires the same conditions stated in Collorary 4 in the paper: the unfavorable scenario has excess inventory under full information disclosure, which motivates the seller to use information pooling to attract more incoming customers; the favorable scenario can sell out all inventory under aggregate information disclosure, which ensures the seller does not lose profit. We present the condition in the following proposition. We can see our main result sustains regardless of whether the search cost is constant or is endogenously related with customers' tastes.

Proposition 2 (Heterogeneous Customers with Heterogeneous Search Cost)

- *When* $\theta_L - p_L \geq \theta_H - p_H$*, aggregate inventory information disclosure is the unique equilibrium if and only if* $\max\{\left(\theta_L - \frac{\hat{c}N}{N-qM/(1-p_H/\theta_H)}\right)\left(1 - \frac{(1-q)M}{N-qM/(1-p_H/\theta_H)}\right), \left(\theta_L - \frac{\hat{c}-\theta_H q}{1-q}\right)(1-\frac{M}{N}) - p_H\frac{q}{1-q}\} \leq p_L \leq \max\{\left(\theta_L - \frac{2q\hat{c}N}{N-(1-q)M/(1-p_H/\theta_H)}\right)\left(1 - \frac{qM}{N-(1-q)M/(1-p_H/\theta_H)}\right), (\theta_H + \theta_L)(1 - M/N) - p_H - 2(1-M/N)\hat{c}\}$*. Otherwise, full information disclosure is the unique equilibrium.*
- *When* $\theta_L - p_L < \theta_H - p_H$*, aggregate inventory information disclosure is the unique equilibrium if and only if* $\left(\theta_L - \frac{\hat{c}-\theta_H q}{1-q}\right)(1-\frac{M}{N})\frac{1-q}{q} - p_H\frac{1-q}{q} + (\theta_H - \theta_L)\frac{1-2q}{q} \leq p_L \leq (\theta_H + \theta_L)(1-M/N) - p_H - 2(1-M/N)\bar{c}$*. Otherwise, full information disclosure is the unique equilibrium.*

3 Heterogeneous Customers: Property Analysis

Recall that for homogeneous customers who are identical in their valuations of the products, the aggregate-disclosure equilibrium occurs within a small price range, when customers incur a small search cost, because nearly all customers in the market visit the store, and thus aggregate disclosure is less effective in affecting consumer demand and stock levels. As the search cost decreases, the price region for aggregate information disclosure also decreases; that is, aggregate disclosure is less likely to be an equilibrium.

In contrast, for heterogeneous customers with diverse valuations for the products, aggregate disclosure sustains as an equilibrium within a large price range even under a small search cost. For aggregate disclosure to be an equilibrium, leftover inventories should exist under the unfavorable inventory scenario. Heterogeneous customer valuation implies that some customers will not switch to purchase their undesired product even if the undesired one is available, which then increases the possibility of leftover inventories compared to the case of homogeneous customers. Consequently, given fixed inventory levels, diverse customer valuations provide the unfavorable inventory scenario a greater incentive to pool information, which leads to a larger price range under which the seller discloses aggregate inventory information in equilibrium.

We state in the following proposition that not only is aggregate disclosure an equilibrium under a small search cost, but the likelihood of aggregate disclosure in equilibrium increases as the search cost decreases.

Proposition 3 *Under* $\theta_L/\theta_H > q/(q+1)$ *and* $\theta_L(N-M) \geq \theta_H M$*, disclosing aggregate inventory information is more likely to be an equilibrium as the traveling cost decreases.*

When customers' tastes are diverse, the low-valuation customers will not switch to purchasing the undesired products, no matter what. As a result, even if all customers visit the store under a small search cost, there will not be enough switchable customers who are willing to purchase the non-preferred products and some inventory will therefore be left over. This implies that the seller in the unfavorable scenario strictly prefers to increase in-store demand, even with a small search cost. Therefore, the aggregate information disclosure region is strictly positive even when the search cost is sufficiently small.

Surprisingly, Proposition 3 also shows that as the search cost decreases, aggregate disclosure is more likely to be an equilibrium. When it costs less to travel to the store, more customers visit the store, and the competition for obtaining the preferred product becomes more intense. More switchable customers are then compelled to switch to the less-preferred product, and the increased sales of less-preferred products due to aggregate disclosure are larger in the favorable inventory scenario. It becomes easier for the seller in the favorable scenario to induce enough demand than for the seller in the unfavorable scenario to attract customers. Thus, the incentive for the seller in the favorable scenario to reduce the penalty cost is higher than the incentive for the seller in the unfavorable scenario to boost demand, i.e., the price adjustment (due to search cost) in condition (B) dominates the price adjustment in condition (A). Therefore, the price region of the aggregate disclosure equilibrium increases, and the seller is more likely to pool information.

However, it is worth noting that this result does not imply that the seller has a stronger incentive to pool information as the search cost decreases. Actually, in this situation, the seller's utility difference between aggregate disclosure and full disclosure decreases. The fact that aggregate disclosure is more likely to be the optimal equilibrium means that this strategy is likely to occur under a large price region.

The use of the technical conditions $\theta_L/\theta_H > q/(q+1)$ and $\theta_L(N-M) \geq \theta_H M$ aids in deriving the comparative analysis. However, the fact that aggregate disclosure becomes less likely as the traveling cost decreases is neither driven or restricted by these technical conditions.

4 Heterogeneous Customers: Allocation Rule Analysis

In the paper, we have focused on the situation in which heterogeneous customers arrive at the store in a random sequence, so that the probability of obtaining the desired product upon arrival is independent of consumer valuations. In reality, customers with different valuations might have different preferences for an early visit or a later visit. In this section, we consider the efficient and inefficient allocation rules studied in the revenue management literature (e.g., Talluri and van Ryzin 2006 and Porteus et al. 2010). Efficient allocation, which is also called high-to-low arrivals, means that higher-valuation customers visit the store and obtain the product first. In contrast, inefficient allocation, which is also called low-to-high arrivals, means that the lower-valuation customers visit the store and obtain the product first. The rest of the assumptions are the same as in Section 5.1.

Recall that under the random allocation rule, each incoming customer has an equal probability of obtaining the product. This means that a customer visits the store as long as he has a positive expected utility, even though he might fail to purchase any products in the store. The key difference in the efficient and inefficient allocation rules is that there is a perfect product-to-customer relation. Under full information disclosure, customers know the exact purchasing sequence and the exact type of product they can obtain at the store based on their types. Customers visit the store only if they can purchase the desired product for sure and the utility is positive. Hence, when the seller fully discloses information, the incoming demand is the same as the inventory sold.

As a result, under full disclosure, the favorable scenario does not incur a shortage penalty cost in both efficient and inefficient rationing rules. Thus, the seller does not have an incentive to attract more customers under these allocation rules, and hence, is indifferent between aggregate disclosure and full disclosure. Condition (B) becomes: in the favorable inventory scenario, the same number of incoming customers is attracted under full disclosure as under aggregate disclosure, which requires that the last arriving customer under full information receive a positive expected utility under aggregate information. Condition (A) remains the same, in that the seller in the unfavorable scenario overstocks under full disclosure and any new customers who arrive due to information pooling should be switchable. These two conditions work for both efficient and inefficient allocation rules. The following proposition summarizes the above conditions.

Proposition 4

*Under the **efficient** allocation rule, disclosing aggregate inventory information is an equilibrium if and only if* $(1-qM/N)\theta_H - c < p_H < \min\{(1-qM/N)\theta_H,\ (1-qM/N)(\theta_H+\theta_L) - p_L - 2c\}$ *is satisfied, or* $(1-qM/N)\theta_H \leq p_H \leq (1-(1-q)M/N)\theta_H$ *and* $p_L < (1-qM/N)\theta_L - 2c$ *are satisfied.*

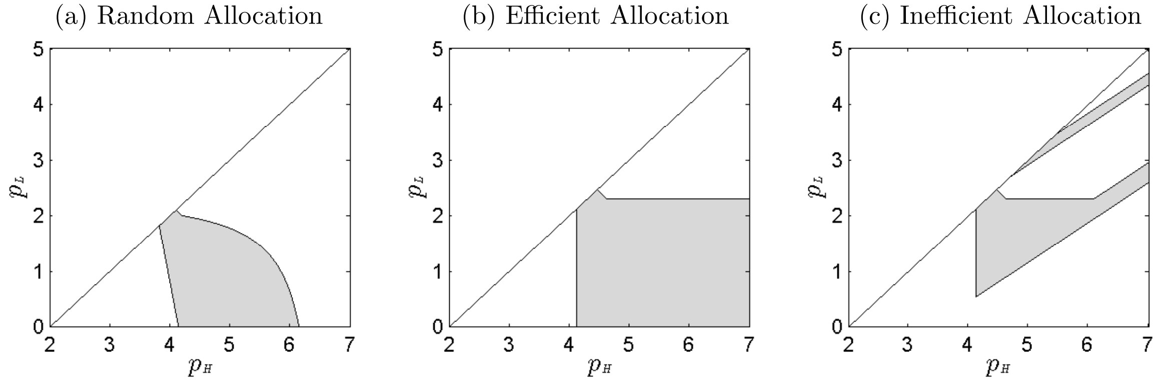
*Under the **inefficient** allocation rule, for* $c/\theta_H > (1-q)M/N$ *and* $p_H < (1-qM/N)\theta_H$*,*[1] *disclosing aggregate inventory information is an equilibrium if and only if* $(1-qM/N)\theta_H - c < p_H < \min\{(p_L + 2c)\theta_H/\theta_L + (2q-1)\theta_H M/N,\ (\theta_H+\theta_L)(1-qM/N) - p_L - 2c\}$ *is satisfied.*

Figure 1 illustrates the price region for aggregate inventory disclosure in equilibrium under the random allocation in panel (a), the efficient allocation in panel (b), and the inefficient allocation in panel (c). Aggregate information disclosure is an equilibrium even when prices are relatively high, as shown in panels (b) and (c). This is because, for both efficient and inefficient allocation rules, customers do not have purchasing uncertainties under full disclosure, and the last incoming customer in the favorable scenario can have a positive utility even with high prices. After information pooling, this last customer might face the unfavorable scenario and end up purchasing the undesired product. Nonetheless, his expected utility can still be positive. Thus, incoming demand is not affected by information pooling, and the seller's desire in the favorable scenario not to lose sales in condition (B) can be satisfied under high prices. Under the random allocation rule, however, every incoming customer has a zero utility. Pooling information will certainly diminish demand in the favorable scenario, and only a low price can guarantee understocking. As a consequence, an aggregate sharing equilibrium exists in a higher price range under both efficient and inefficient allocation rules.

Our second observation is that under the inefficient allocation rule, the aggregate sharing equilibrium can occur when both p_L and p_H are relatively high. Because lower-type customers visit the store first, the last incoming customer is a higher type who is more likely to be switchable and receives a positive utility under aggregate information disclosure. This fact enhances the seller's ability to up-sell or cross-sell the alternative products, sustaining pooling information as an equilibrium under a higher price range.

[1]This condition is a technical condition that we impose to make the exposition simple. However, the complete characterization of the price region for the aggregate inventory information disclosure is given in the appendix. Furthermore, Figure 1 depicts the complete characterization of the price region.

Figure 1: The aggregate disclosure strategy under random, efficient and inefficient allocation rules. Parameter values are $\theta_L = 5$, $\theta_H = 7$, $N = 10$, $M = 3$, $\beta = 0.5$, $q = 0.9$ and $c = 0.5$.



5 Proof Appendix

Proof of Proposition 1: When $qM < N < M$, the marginal utility equation (1) in the paper becomes $(1-q)M/N_u^{Full}(\theta_L - p_L) + (1-(1-q)M/N_u^{Full})(\theta_H - p_H) = c$. Under aggregate information disclosure, since all customers visit the store, their utility can be positive. The marginal utility equation (2) in the paper becomes $M/N^{Agg}(\beta(1-q)+(1-\beta)q)(\theta_L - p_L) + (1 - M/N^{Agg}(\beta(1-q)+(1-\beta)q))(\theta_H - p_H) \geq c$. We solve the two conditions and obtain

$$\theta_H + \frac{(1-q)M(\theta_L - p_L) - Nc}{N-(1-q)M} < p_H < \theta_H + \frac{(q(1-\beta)+(1-q)\beta)\,M(\theta_L - p_L) - Nc}{N - M(q(1-\beta)+(1-q)\beta)}.$$

When $(1-q)M < N < qM$, the marginal utility equation (1) in the paper becomes $(1-q)M/N_u^{Full}(\theta_L - p_L) + (1-(1-q)M/N_u^{Full})(\theta_H - p_H) = c$. Under aggregate information disclosure, since all customers visit the store, their utility can be positive. The marginal utility equation (2) in the paper becomes $\beta\left((1-q)M/N^{Agg}(\theta_L - p_L) + qM/N^{Agg}(\theta_H - p_H)\right) + (1-\beta)(\theta_L - p_L) = c$. We solve the two conditions and obtain

$$\theta_H + \frac{(1-q)M(\theta_L - p_L) - Nc}{N-(1-q)M} < p_H < \theta_H + \frac{(N+\beta(1-q)M - \beta N)(\theta_L - p_L) - Nc}{Mq\beta}. \square$$

Proof of Proposition 2: An in-store customer with type v that satisfies $\theta_H v - p_H \geq 0$ switches to purchasing the undesired product, if the desired item is out of stock. The type v customer with $\theta_H v - p_H < 0$ will not purchase the undesired item no matter what.

We first consider the case where every customer prefers the low quality product, i.e., $\theta_L - p_L > \theta_H - p_H$. When $x \leq p_H/\theta_H$, the marginal equation (1) becomes $(\theta_L x - p_L)\frac{(1-q)M}{(1-x)N} = x\hat{c}$. Condition (A) requires that in the unfavorable scenario, incoming demand for the undesired product under full information is less than its inventory, i.e., $(1 - \frac{(1-q)M}{(1-x)N})(1 - p_H/\theta_H)N < qM$. The price boundary in condition (A) becomes

$$\underline{p_L}(p_H) = \left(\theta_L - \frac{\hat{c}N}{N - qM/(1-p_H/\theta_H)}\right)\left(1 - \frac{(1-q)M}{N - qM/(1-p_H/\theta_H)}\right).$$

When $x \geq p_H/\theta_H$, condition (A) becomes $x > 1 - M/N$. The marginal equation (1) becomes $(\theta_L x - p_L)\frac{(1-q)M}{(1-x)N} + (\theta_H x - p_H)(1 - \frac{(1-q)M}{(1-x)N}) = x\hat{c}$. The price boundary in condition (A) is

$$\underline{p_L}(p_H) = \left(\theta_L - \frac{\hat{c} - \theta_H q}{1-q}\right)(1 - \frac{M}{N}) - p_H\frac{q}{1-q}.$$

When $y \le p_H/\theta_H$, the marginal equation (2) becomes $(\theta_L y - p_L)\frac{(1-q)M}{(1-y)N} + (\theta_L y - p_L)\frac{qM}{(1-y)N} = 2y\hat{c}$. Condition (B) requires that in the favorable scenario, incoming demand for the desired product exceeds its inventory under aggregate information, i.e., $(1 - \frac{qM}{(1-y)N})(1 - p_H/\theta_H)N > (1-q)M$. The price boundary in condition (B) becomes

$$\bar{p_L}(p_H) = \left(\theta_L - \frac{2q\hat{c}N}{N - (1-q)M/(1-p_H/\theta_H)}\right)\left(1 - \frac{qM}{N - (1-q)M/(1-p_H/\theta_H)}\right).$$

When $y \ge p_H/\theta_H$, condition (B) becomes $y < 1 - M/N$. The marginal equation (2) becomes $(\theta_L y - p_L)\frac{(1-q)M}{(1-y)N} + (\theta_H y - p_H)\frac{qM}{(1-y)N} + (\theta_L y - p_L)\frac{qM}{(1-y)N} + (\theta_H y - p_H)\frac{(1-q)M}{(1-y)N} = 2y\hat{c}$. The price boundary in condition (B) is

$$\bar{p_L}(p_H) = (\theta_H + \theta_L)(1 - M/N) - p_H - 2(1 - M/N)\hat{c}.$$

Next, we consider the case where some customers with high type prefer the high quality product, i.e., $\theta_L - p_L < \theta_H - p_H$. When the incoming customer with the lowest type prefers the low quality product, since her type is above p_H/θ_H, the above analysis remains and the conditions obtained above hold. When all incoming customer prefer the high quality product, the high quality product becomes the favorable item for every customer, and thus, we have the symmetric analysis. The price boundary in condition (A) for the unfavorable scenario becomes

$$\underline{p_L}(p_H) = \left(\theta_L - \frac{\hat{c} - \theta_H q}{1-q}\right)(1 - \frac{M}{N})\frac{1-q}{q} - p_H\frac{1-q}{q} + (\theta_H - \theta_L)\frac{1-2q}{q}.$$

The price boundary in condition (B) for the favorable scenario becomes

$$\bar{p_L}(p_H) = (\theta_H + \theta_L)(1 - M/N) - p_H - 2(1 - M/N)\bar{c}.\square$$

Proof of Proposition 3: **Region (a):** $x \le p_H/\theta_H$ **and** $y \le p_H/\theta_H$. This condition is equivalent to $c \le (1-q)M(p_H\theta_L - p_L\theta_H)(\theta_H - p_H)^{-1}N^{-1}$. The marginal utility equation (1) yields $x = (cN/(1-q)M + p_L)/(cN/(1-q)M + \theta_L)$. Condition (A) is

$$\underline{p_L}(p_H) = \theta_L - (1 - p_H/\theta_H)\frac{cN + (1-q)M\theta_L}{N(1 - p_H/\theta_H) - qM}. \tag{1}$$

The marginal utility equation (2) yields $y = (2cN/M + p_L)/(2cN/M + \theta_L)$. Condition (B) is

$$\bar{p_L}(p_H) = \theta_L - (1 - p_H/\theta_H)\frac{2qcN + qM\theta_L}{N(1 - p_H/\theta_H) - (1-q)M}. \tag{2}$$

We denote the interception of equation (1) with $p_H - p_L = \theta_H - \theta_L$ as $(\tilde{p}_H^U(\theta_H - \theta_L), \tilde{p}_L^U(\theta_H - \theta_L))$, and the interception of equation (2) with $p_H - p_L = \theta_H - \theta_L$ as $(\tilde{p}_H^F(\theta_H - \theta_L), \tilde{p}_L^F(\theta_H - \theta_L))$, where

$$\begin{aligned}
\tilde{p}_H^U(\theta_H - \theta_L) &= -c + \theta_H - \frac{M\theta_L}{N} - \frac{qM(\theta_H - \theta_L)}{N}, \\
\tilde{p}_H^F(\theta_H - \theta_L) &= -2qc + \theta_H - \frac{M\theta_H}{N} + \frac{qM(\theta_H - \theta_L)}{N}.
\end{aligned}$$

The distance of $(\tilde{p}_H^F(\theta_H - \theta_L), \tilde{p}_L^F(\theta_H - \theta_L))$ and $(\tilde{p}_H^U(\theta_H - \theta_L), \tilde{p}_L^U(\theta_H - \theta_L))$ is $\sqrt{2}(\tilde{p}_H^F(\theta_H - \theta_L) - \tilde{p}_H^U(\theta_H - \theta_L))$. Since $\partial(\tilde{p}_H^F(\theta_H - \theta_L) - \tilde{p}_H^U(\theta_H - \theta_L))/\partial c = 1 - 2q < 0$, the distance of the two conditions' projection on $p_H - p_L = \theta_H - \theta_L$ decreases as the search cost increases.

We denote the interception with $p_H - p_L = d$ as $(\tilde{p}_H^i(d), \tilde{p}_L^i(d))$, where $i \in \{U, F\}$. The distance of $(\tilde{p}_H^F(d), \tilde{p}_L^F(d))$ and $(\tilde{p}_H^U(d), \tilde{p}_L^U(d))$ is $\sqrt{2}(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))$. The area of aggregate information sharing integrates the distance $\sqrt{2}(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))$ along $d \in [\theta_H - \theta_L, \bar{p_H}(0)]$ and the distance $\sqrt{2}(\tilde{p}_H^F(d) - d)$ along $d \in [\bar{p_H}(0), \underline{p_H}(0)]$. As c decreases, $\underline{p_H}(0)$ and $\bar{p_H}(0)$ increases. The interceptions satisfy

$$
\begin{aligned}
f_1 &\equiv (\tilde{p}_H^U(d) - d - \theta_L)(N(\theta_H - \tilde{p}_H^U(d)) - qM\theta_H) + (\theta_H - \tilde{p}_H^U(d))(cN + (1-q)M\theta_L) = 0, \quad (3)\\
f_2 &\equiv (\tilde{p}_H^F(d) - d - \theta_L)(N(\theta_H - \tilde{p}_H^F(d)) - (1-q)M\theta_H) + (\theta_H - \tilde{p}_H^F(d))(2qcN + qM\theta_L) = 0.
\end{aligned}
$$

The derivative becomes

$$
\frac{\partial(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))}{\partial c} = \frac{cN + ((1-q)\theta_L + q\theta_H)M}{\varepsilon N + cN + ((1-q)\theta_L + q\theta_H)M} - 2q\frac{2qcN + ((1-q)\theta_H + q\theta_L)M}{\varepsilon N + 2qcN + ((1-q)\theta_H + q\theta_L)M},
$$

where $\varepsilon = d - (\theta_H - \theta_L) \geq 0$. Since $\theta_H > \theta_L$ and $q > 1/2$, then $(1-q)\theta_H + q\theta_L \leq (1-q)\theta_L + q\theta_H$. If $cN + ((1-q)\theta_L + q\theta_H)M \leq 2qcN + ((1-q)\theta_H + q\theta_L)M$, since $2q\varepsilon N + 2qcN + 2q((1-q)\theta_L + q\theta_H)M \geq \varepsilon N + 2qcN + ((1-q)\theta_H + q\theta_L)M$, we have $\partial(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))/\partial c \leq 0$. Otherwise, the gap is $(2q-1)(cN(2q+1) - q\theta_H M + (q+1)\theta_L M)$. Given the condition $\theta_L/\theta_H > q/(q+1)$, we have $\partial(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))/\partial c \leq 0$.

The distance of the two conditions' projection on $p_H - p_L = d$ increases as the search cost decreases. Since $\partial \tilde{p}_H^F(d)/\partial c < 0$, the distance $\sqrt{2}(\tilde{p}_H^F(d) - d)$ increases as the search cost decreases. The aggregate information sharing area integrates the distance long $d \in [\theta_H - \theta_L, \underline{p_H}(0)]$. Since $\underline{p_H}(0)$ decreases in c, the aggregate disclosure area increases as c decreases.

Region (b): $x > p_H/\theta_H$ **and** $y \leq p_H/\theta_H$, which is equivalent to $c \leq M(p_H\theta_L - p_L\theta_H)(\theta_H - p_H)^{-1}(2N)^{-1}$ and $p_H < (1 - M/N)\theta_H$. If $x > p_H/\theta_H$, condition (A) becomes $x > 1 - M/N$. The marginal utility equation (1) yields $\underline{p_L}(p_H) = -p_H\frac{q}{1-q} - (\theta_H - \theta_L)(1 - \frac{M}{N}) - \frac{c}{1-q} + \frac{\theta_H}{1-q}(1 - \frac{M}{N})$. Under the condition $x > p_H/\theta_H$, $p_H < \theta_H(1 - M/N)$. Condition (A) in region (b) is $\underline{p_L}(\theta_H - \theta_H M/N) = \theta_L(1 - M/N) - c/(1-q)$. Region (b) corresponds to $d \in [\theta_H - \theta_L, (\theta_H - \theta_L)(1 - M/N) + c/(1-q)]$. The interception of condition (A) with $p_H - p_L = d$ becomes $\tilde{p}_H^U(d) = -c + (1-q)d - (M(1-q)(\theta_H - \theta_L) + N\theta_H)(M^{-1} - N^{-1}) - \theta_H(N-M)^2/NM$. The interception of condition (B) is the same as in equation (3). The derivative becomes

$$
\frac{\partial(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))}{\partial c} = \frac{\partial f_1/\partial c}{\partial f_1/\partial p_H} - \frac{\partial f_2/\partial c}{\partial f_2/\partial p_H} = 1 - 2q\frac{2qcN + ((1-q)\theta_H + q\theta_L)M}{\varepsilon N + 2qcN + ((1-q)\theta_H + q\theta_L)M}. \quad (4)
$$

The above equation has the same sign as $\varepsilon N - (2q-1)2qcN - (2q-1)((1-q)\theta_H + q\theta_L)M$. Since $\varepsilon \leq -(\theta_H - \theta_L)M/N + c/(1-q)$, we have

$$
\varepsilon N - (2q-1)2qcN - (2q-1)((1-q)\theta_H + q\theta_L)M \leq \frac{cN}{1-q} - \theta_H M - (2q-1)\left(2qcN + (1-q)(\theta_H - \theta_L)M\right).
$$

Since $\underline{p_L}(\theta_H - \theta_H M/N) \geq 0$, we have $c/(1-q) \leq \theta_L(1 - M/N)$. Given $\theta_L(N-M) \geq \theta_H M$, equation (4) is negative. Thus, the aggregate disclosure region decreases in c in Region (b).

Region (c): $x > p_H/\theta_H$ **and** $y > p_H/\theta_H$. If $y > p_H/\theta_H$, condition (B) becomes $y \leq 1 - M/N$. The marginal utility Equation (1) yields $\underline{p_L}(p_H) = -p_H - 2c + (\theta_L + \theta_H)(1 - M/N)$. The interception of condition (B) with $p_H - p_L = d$ is $\tilde{p}_H^F(d) = -c + \frac{d}{2} + \frac{(\theta_L + \theta_H)(1 - M/N)}{2}$. The interception of condition (A) is the same as in Region (b). The derivative is

$$
\frac{\partial(\tilde{p}_H^F(d) - \tilde{p}_H^U(d))}{\partial c} = 0.
$$

Thus, the aggregate disclosure region remains constant in Region (c). □

Proof of Proposition 4: We define two types of customers. Customers beyond type $(p_H+c)/\theta_H$ receive a positive value from purchasing the unfavorable product. These customers will visit the store, if they know for sure that they will obtain the unfavorable product. We refer to them as *unfavo-effective*. Let v_H denote $(p_H+c)/\theta_H$ and let v_L denote $(p_L+c)/\theta_L$. Since $p_L/\theta_L < p_H/\theta_H$, we have $v_L < v_H$. Customers between v_L and v_H receive a positive value from purchasing their favorable products and a negative value from purchasing their unfavorable product. They choose not to visit the store, if they will obtain the unfavorable product for sure. We refer to these customers as *unfavo-ineffective*.

Efficient allocation. The unfavorable scenario overstocks under full disclosure if and only if $N(1-v_H) < M$. If $N(1-v_H) \geq qM$, both inventory scenarios can sell $N(1-v_H)$ products under full disclosure, and type v_H customer purchases the unfavorable products. This implies that only customers strictly above v_H will visit the store under aggregate information. If $N(1-v_H) < qM$, the favorable scenario attracts more sales than the unfavorable scenario under full information, because it induces the unfavo-ineffective customers. If $N(1-p_H/\theta_H) < (1-q)M$, customers who cannot purchase the favorable product will not switch to the unfavorable product and the unfavorable scenario cannot increase sales under aggregate disclosure. Hence, aggregate information increases sales for the unfavorable scenario if and only if $N(1-v_H) < qM$ and $N(1-p_H/\theta_H) > (1-q)M$.

The favorable scenario does not lose sales under aggregate information, if the last incoming customer under full information still visits the store. The last potential incoming customer has type $\max\{1-qM/N, v_L\}$. It must be that $1-qM/N > v_L$; otherwise, type v_L customer will not visit the store under aggregate information, which violates condition (B). Therefore, the last incoming customer in the favorable scenario under full information has type $1-qM/N$.

The last incoming customer should obtain a positive utility under aggregate information.

- If $1-qM/N > p_H/\theta_H$, condition (B) becomes $(1-qM/N)\theta_L - p_L + (1-qM/N)\theta_H - p_H > 2c$. We have $1-qM/N > \max\{(2c+p_H+p_L)/(\theta_H+\theta_L),\, p_H/\theta_H\}$.
- If $1-qM/N \leq p_H/\theta_H$, condition (B) becomes $(1-qM/N)\theta_L - p_L > 2c$. We have $(2c+p_L)/\theta_L < 1-qM/N \leq p_H/\theta_H$.

Inefficient allocation. The complete characterization of the price region is as follows

(i) for $p_H < (1-qM/N)\theta_H$, $(1-qM/N)\theta_H - c < p_H < \min\{(p_L+2c)\theta_H/\theta_L + (2q-1)\theta_H M/N,$ $(\theta_H+\theta_L)(1-qM/N) - p_L - 2c\}$ is satisfied;

(ii) for $(1-qM/N)\theta_H \leq p_H < (1-(1-q)M/N)\theta_H - c$, $p_H < (p_L+2c)\theta_H/\theta_L + (2q-1)\theta_H M/N$ and $p_L < (1-qM/N)\theta_L - 2c$ are satisfied, or $(p_L+c)\theta_H/\theta_L + (1-q)\theta_H M/N - c < p_H < (p_L+c)(\theta_H/\theta_L+1) + (1-q)(\theta_H+\theta_L)M/N - p_L - 2c$ are satisfied;

(iii) for $p_H \geq (1-(1-q)M/N)\theta_H - c$, $(p_L+2c)\theta_H/\theta_L + (2q-1)\theta_H M/N - c < p_H < (p_L+2c)\theta_H/\theta_L + (2q-1)\theta_H M/N$ is satisfied, or $(p_L+c)\theta_H/\theta_L + (1-q)\theta_H M/N - c < p_H < (p_L+c)(\theta_H/\theta_L+1) + (1-q)(\theta_H+\theta_L)M/N - p_L - 2c$ is satisfied.

In the unfavorable scenario, if the unfavo-ineffective demand is less than the favorable product inventory, all unfavo-effective customers visit the store and the unfavorable scenario sells off, which does not provide the unfavorable scenario any incentive to pool information. As a result, $(v_H - v_L)N > (1-q)M$ is required. The unfavorable scenario should overstock under full disclosure, and thus we need $(1-v_H)N < qM$.

Under aggregate information, $(1-q)M$ lowest unfavo-ineffective customers will visit the store for sure. Customers above might face unfavorable products in the unfavorable scenario. We denote the upper and lower bound of such customers by $\underline{v}$ and $\overline{v}$.

- If $p_H/\theta_H \le \underline{v}$, consumer $\underline{v}$'s utility is $(\theta_L \underline{v} - p_L)/2 + (\theta_H \underline{v} - p_H)/2 - c = 0$ and $\underline{v} = (2c + p_H + p_L)/(\theta_H + \theta_L)$. The new arrivals can switch to the unfavorable product and the unfavorable scenario can increase the profit. Condition (A) becomes $p_H/\theta_H < (2c + p_H + p_L)/(\theta_H + \theta_L)$.

- If $p_H/\theta_H > \underline{v}$, consumer $\underline{v}$'s utility is $(\theta_L \underline{v} - p_L)/2 - c = 0$ and $\underline{v} = (2c + p_L)/\theta_L$. At least some new arrivals are switchable. We need $\overline{v} > p_H/\theta_H$, where $\overline{v} = \max\{(2c + p_L)/\theta_L, v_L + (1-q)M/N\} + (2q-1)M/N$. Condition (A) becomes $(2c + p_L)/\theta_L < p_H/\theta_H < \max\{(2c + p_L)/\theta_L, v_L + (1-q)M/N\} + (2q-1)M/N$.

We next analyze the favorable scenario's incentive in condition (B).

- If the unfavo-ineffective demand is smaller than the favorable product, $N(v_H - v_L) < qM$, and the total incoming customers are less than the total inventory, $(1 - v_L)N < M$, then all customers visit the store, and condition (B) becomes $N \min\{(2c + p_H + p_L)/(\theta_H + \theta_L), (p_L + 2c)/\theta_L\} < v_L N + (1-q)M$.

- If $N(v_H - v_L) < qM$ and $(1 - v_L)N \ge M$, condition (B) becomes $N \min\{(2c + p_H + p_L)/(\theta_H + \theta_L), (p_L + 2c)/\theta_L\} < N - qM$.

- If $N(v_H - v_L) \ge qM$ and the unfavo-effective customers are less than the unfavorable product, $(1 - v_H)N < (1-q)M$, the same amount of unfavo-ineffective customers visit the store. Condition (B) becomes $N \min\{(2c + p_H + p_L)/(\theta_H + \theta_L), (p_L + 2c)/\theta_L\} < v_H N - (2q-1)M$.

- If $N(v_H - v_L) \ge qM$ and $(1 - v_H)N \ge (1-q)M$, condition (B) becomes $N \min\{(2c + p_H + p_L)/(\theta_H + \theta_L), (p_L + 2c)/\theta_L\} < N - qM$. $\square$

References

Chen, K.-Y., M. Kaya, O. Özer. 2008. Dual sales channel management with service competition. *Manufacturing & Service Oper. Management* **10**(4) 654–675.

Moon, K., K. Bimpikis, H. Mendelson. 2015. Randomized markdowns and online monitoring. *Available at SSRN 2589386* .

Porteus, E. L., H. Shin, T. I. Tunca. 2010. Feasting on leftovers: Strategic use of shortages in price competition among differentiated products. *Manufacturing & Service Oper. Management* **12**(1) 140–161.

Talluri, K. T, G. J van Ryzin. 2006. *The Theory and Practice of Revenue Management*, vol. 68. New York, NY: Springer.